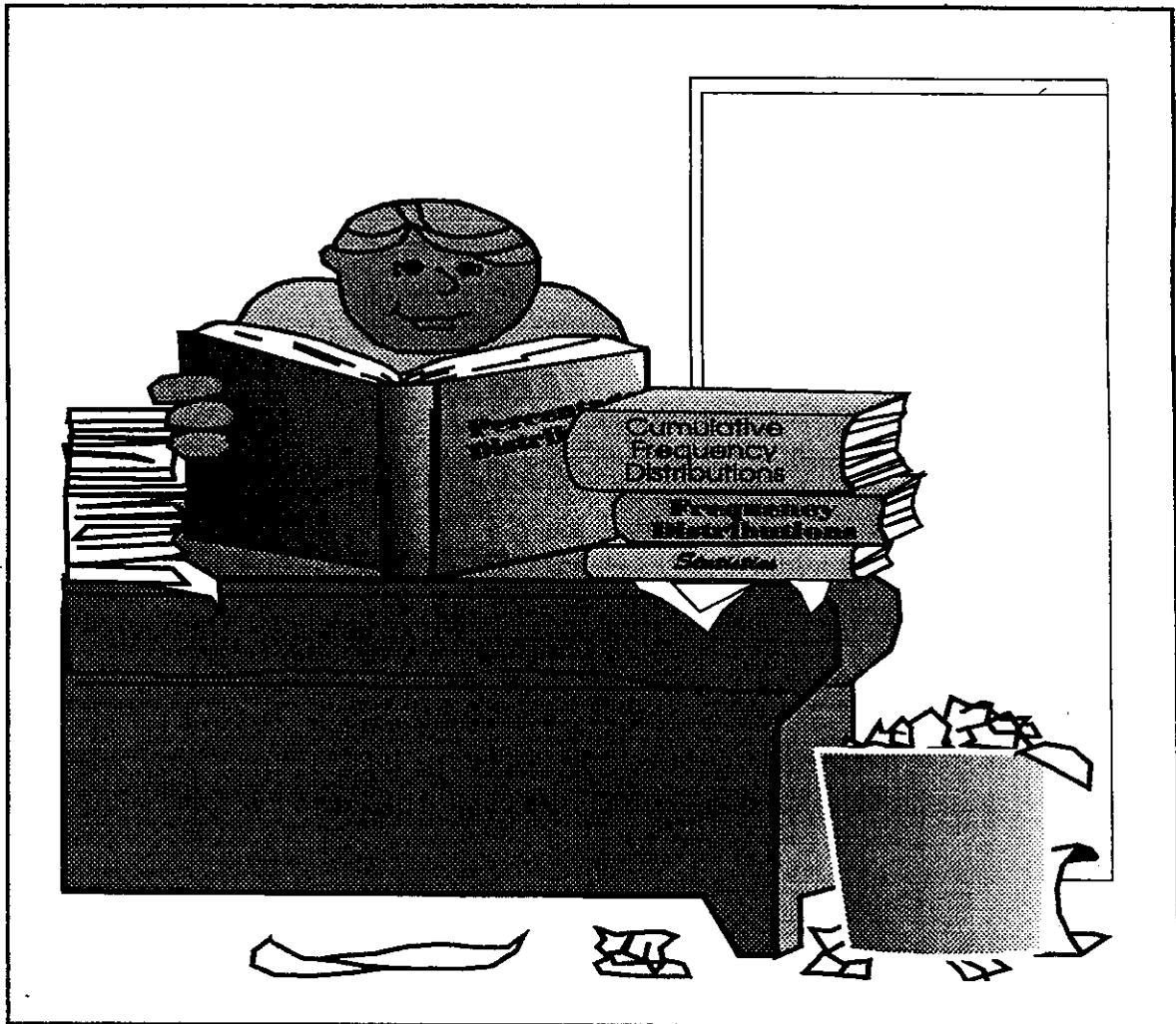## Lesson 6

Cumulative Percentage Distributions

Questions To Consider

- What are cumulative percentage distributions?
- How do you read and create cumulative distribution graphs and tables?

Key Terms

- Cumulative percentage distributions
- Cumulative percentage distribution table
- Cumulative percentage histogram
- Cumulative relative-frequency distributions
- Cumulative relative-frequency polygon
- Percentage ogive

"Running Total" Of Percentages

You'll be happy to hear that this is the last of the four "distribution" lessons. Even better news is that **cumulative percentage distributions** are sort of a combination of cumulative frequency distributions (Lesson 4) and percentage distributions (Lesson 5). Just take the characteristics of each of those, combine them, and—voila—you have it! For the record—just as percentage distributions can be referred to as relative-frequency distributions, so can cumulative percentage distributions be referred to as **cumulative relative-frequency distributions**.

Remember the cumulative frequency distributions from Lesson 4? They used sort of "running totals" that indicated amounts that were "less than" a class limit. And, in Lesson 5, you saw tables that showed percentages, rather than actual numbers, of occurrences in a set. Cumulative percentage distributions are a combination of those two. Take, for example, Table 6-1.

Cumulative percentage distributions display the "running totals" of cumulative frequency distributions in a percentage distribution format.

Table 6-1. Percentage Distribution

Income level	Percentage of households				
	1975	1976	1977	1978	1979
Under $5000	19.9	17.8	16.5	14.7	13.2
$ 5,000-$ 9,999	22.4	21.4	20.2	18.4	16.4
$10,000-$ 14,999	20.5	19.1	18.0	16.6	15.9
$15,000-$ 19,999	15.9	16.4	15.6	15.3	14.0
$20,000-$ 24,999	9.6	10.7	11.5	12.2	12.4
$25,000-$ 34,999	7.8	9.5	11.5	13.8	15.6
$35,000-$ 49,999	2.7	3.5	4.5	6.0	8.3
$50,000-$100,000	1.2	1.6	2.2	3.0	4.2

Note that the body of the table contains percentages—not actual counts. If you take that table and present it as a "less-than" cumulative (percentage) distribution, you get Table 6-2.

Table 6-2. Cumulative Percentage Distribution

Income level	Percentage of households				
	1975	1976	1977	1978	1979
Under $5000	19.9	17.8	16.5	14.7	13.2
$0-$ 9,999	42.3	39.2	36.7	33.1	29.6
$0-$ 14,999	62.8	58.3	54.7	49.7	45.5
$0-$ 19,999	78.7	74.7	70.3	65.0	59.5
$0-$ 24,999	88.3	85.4	81.8	77.2	71.9
$0-$ 34,999	96.1	94.9	93.3	91.0	87.5
$0-$ 49,999	98.8	98.4	97.8	97.0	95.8
$0-$100,000	100.0	100.0	100.0	100.0	100.0

In 1977, 16.5% of households had an annual income of less than $5,000; 36.7% had an annual income of $9,999 or less; 54.7% had $14,999 or less; 70.3%, $19,999 or less, and so on. Note that in all years, 100% of households had incomes that were less than or equal to the highest stated income.

That was an example of how you take a percentage distribution and create a cumulative percentage distribution. Try the next one yourself. Use Table 6-3 to create Table 6-4.

Look on the next page and check your figures against those in Table 6-4a.

Table 6-3. Percentage Distribution For Do-It-Yourself Cumulative Percentage Distribution

SO₂ level (ppb)	Observations recorded (%)
25- 39	16
40- 54	28
55- 69	16
70- 84	16
85- 99	8
100-114	4
115-129	4
130-144	0
145-159	4
160-174	0
175-189	4

Table 6-4. Do-It-Yourself Cumulative Percentage Distribution

SO₂ level (ppb)	Observations recorded (%)
Less than 40	
55	
70	
85	
100	
115	
130	
145	
160	
175	
190	

Table 6-4a. Do-It-Yourself Cumulative Percentage Distribution (answer)

SO_2 level (ppb)	Observations recorded (%)
Less than 40	16
" 55	44
" 70	60
" 85	76
" 100	84
" 115	88
" 130	92
" 145	92
" 160	96
" 175	96
" 190	100

If your figures were correct, you can skip the next paragraph.

In the "less-than" table (Table 6-4), it's easy to determine the percentage of SO_2 levels that were less than 40 ppb; you can read them directly from Table 6-3. Sixteen percent of the readings were under 40 ppb. To determine the percentage that were less than 55 ppb, you need to add the percentage that were under 40 ppb (16%) to the percentage that were from 40 to 54 ppb (28%), whereupon you get a total of 44%. To determine the percentage that were less than 70 ppb, add the percentage that were from 55 to 69 ppb (16%), the percentage that were from 40 to 54 ppb (28%), and the percentage that were less than 40 ppb (16%). You'll get 60%. You continue that process for the other categories.

A **percentage ogive** (or **cumulative relative-frequency polygon**) is the same as an ogive (Lesson 4), except that it's based on percentages rather than actual frequencies.

An ogive can be created for percentages just as for actual frequencies.

The percentage ogive (Figure 6-1) below is a graphical representation of the table you just filled in.
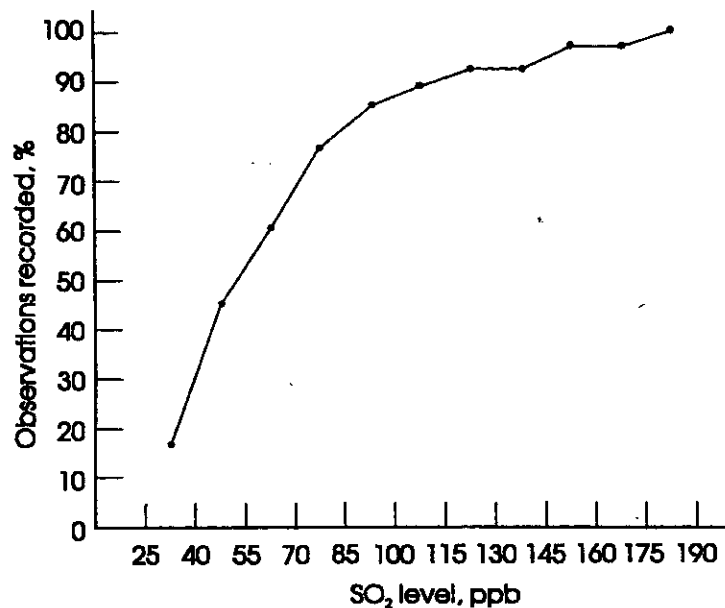

Figure 6-1. Percentage Ogive Based on Table 6-4a

A percentage ogive can be constructed either by using a cumulative percentage histogram and connecting the midpoints of the tops of each bar or by plotting those same points without having first constructed a histogram. The coordinates of each point are the class mark (as the abscissa) and the percentage (as the ordinate) of the interval. For example, the coordinates of the first (left-most) point of Figure 6-1 are (32.5,16); the coordinates of the next point are (47.5,44); and so on for each of the points.

If you have either a **cumulative percentage histogram** or a percentage ogive, you can construct a **cumulative percentage distribution table** from it. The class intervals can either be taken from the widths of the bars of the histograms or be figured out from knowing the class marks of each interval of an ogive. The percentage can be determined either from the height of each bar of a histogram or from the ordinate of each point of an ogive.

Actually, you've done this already (in Lesson 4) for an ogive, but try it here for a percentage ogive. Take the information from Figure 6-2 and construct a cumulative "less-than" percentage distribution table (Table 6-5). The correct data are on the next page.

You can make a cumulative percentage distribution table from either a cumulative percentage histogram or a percentage ogive.
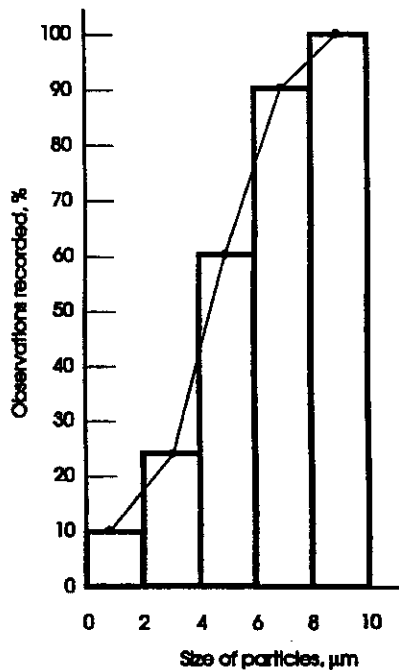

Table 6-5.   Do-It-Yourself Cumulative Percentage Distribution Table

Size of particles (μm)	Observations recorded (%)

Figure 6-2. Cumulative Percentage Histogram And Percentage Ogive For Do-It-Yourself Cumulative Percentage Distribution Table

Table 6-5a. Do-It-Yourself Cumulative Percentage Distribution Table (answer)
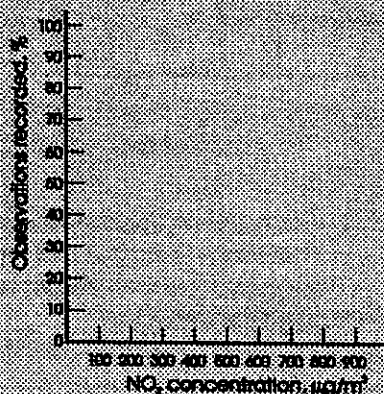
Size of particles (μm)	Observations recorded (%)
0-2"	10
0-4"	25
0-6"	60
0-8"	90
0-10	100

Now that you know the basics about cumulative percentage distributions, try your hand at the following three exercises.

The solutions to these exercises are on the next page.

1. What is a cumulative percentage distribution?

2. Use the cumulative "more-than" percentage distribution table shown below to construct a percentage ogive.

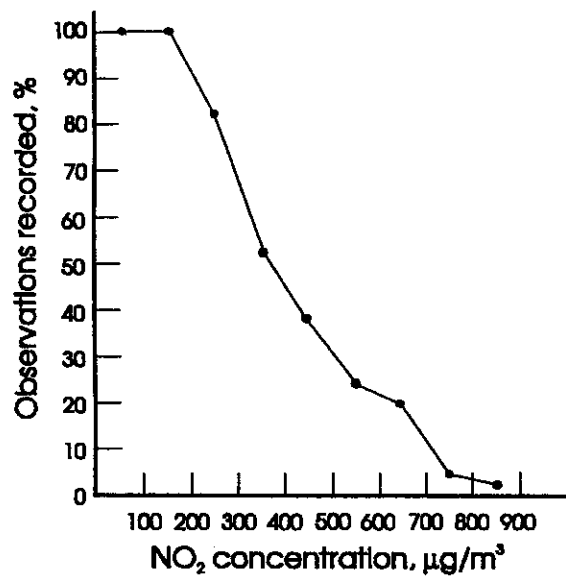| NO₂ concentration (μg/m³)	Observations recorded (%)
0-899.9	100
100-899.9	100
200-899.9	82
300-899.9	62
400-899.9	36
500-899.9	24
600-899.9	20
700-899.9	4
800-899.9	2

3. Use the cumulative "more-than" percentage distribution table in question #2 to complete the (non-cumulative) percentage distribution table below.

NO₂ concentration (μg/m³)	Observations recorded (%)
0- 99.9	
100-199.9	
200-299.9	
300-399.9	
400-499.9	
500-599.9	
600-699.9	
700-799.9	
800-899.9	

Solutions

1. A cumulative percentage distribution shows a "running total" of percentages—the percentage of values either above or below a certain value.

2.

3.

NO$_2$ concentration (µg/m^3)	Observations recorded (%)
0- 99.9	0
100-199.9	18
200-299.9	30
300-399.9	14
400-499.9	14
500-599.9	4
600-699.9	16
700-799.9	2
800-899.9	2